...rization p. a_q is the amplitude of the vibration. Then

$$U_{kk'}=-\sum_{q}\sum_{l}\exp[-i(\mathbf{k}-\mathbf{k}'+\mathbf{q})\cdot\mathbf{l}]a_{q,p}\sum_{p}\hat{e}_{q,p}\cdot\mathcal{S}[JS]. \tag{23}$$

The sum over $\mathbf{l}$ yields the condition

$$\mathbf{k}-\mathbf{k}'+\mathbf{q}=\mathbf{K}, \text{ a reciprocal lattice vector, or } =0, \tag{24}$$

and a value N, the number of ions. Since $\mathbf{k}$ and $\mathbf{k}'$ are specified and we have restricted $\mathbf{q}$ to the first Brillouin zone, $\mathbf{q}$ is specified and the sum over $\mathbf{q}$ reduces to a single term. a_q is obtained from the matrix element for a phonon annihilation or creation operator and is given by

$$a_{q,p}=\left[\frac{h}{2NM\omega_{q,p}}\right]^{\frac{1}{2}}\times\begin{cases}[\bar{n}_{q,p}]^{\frac{1}{2}} & \text{annihilation}\\ \text{or} & \\ [\bar{n}_{q,p}+1]^{\frac{1}{2}} & \text{creation,}\end{cases} \tag{25}$$

where M is the mass of the ion and $\omega_{q,p}$ the angular frequency of the phonon $\mathbf{q}$.[21]

$\bar{n}_{q,p}$ the equilibrium number of phonons is given by the factor;

$$\bar{n}_{q,p}=\frac{1}{\exp(\hbar\omega_{q,p}/kT)-1}. \tag{26}$$

In the high temperature limit $\hbar\omega/kT\ll 1$ and $a_{q,p}$ becomes

$$a_{q,p}=\left[\frac{h}{2NM\omega_{q,p}}\frac{kT}{\hbar\omega_{q,p}}\right]^{\frac{1}{2}}=\frac{B^{\frac{1}{2}}}{N\omega_{q,p}}. \tag{27}$$

The constants have been lumped into B; the $1/N$ cancels the N from the sum in Eq. (23).

Then by substituting Eq. (27) into Eq. (23) for $U_{kk'}$ and dropping the sum over p with the understanding that we will consider the polarization that gives the largest contribution to $U_{k,k'}$ we obtain

$$P_{k,\phi}(\theta)=\frac{B}{4\pi^2\hbar|\nabla_k E|_{k'}}\frac{[\{JS\}(\theta)]^2(\hat{e}_{q,p}\cdot\mathcal{S})^2}{\omega_{q,p}^2}. \tag{28}$$

When all the constants are included in D and Eq. (28) is substituted into Eq. (16) we obtain

$$\frac{1}{\tau(\mathbf{k})}=D\int_0^{\pi}d\phi\int_0^{2\pi}d\theta \times\frac{(1-\cos\theta)|\sin\theta|[\{JS\}(\theta)]^2(\hat{e}_{q,p}\cdot\mathcal{S})}{\omega_{q,p}^2|\nabla_k E|_{k'}}. \tag{29}$$

We can now consider the sources of anisotropy in $\tau(\mathbf{k})$. The density of states factor is, strictly speaking, anisotropic since a spherical constant energy surface was assumed in obtaining Eq. (15). If Eq. (15) is still assumed valid for a warped Fermi surface, then $1/|\nabla_k E|_{k'}$ acts as a weighting factor in Eq. (29). The variation of this factor with direction can be estimated using Eq. (5); for a warping typical of the alkali metals, $A_1=-0.02$, $B=B_1=-0.09$, the value of the density of states factor varies by about $\pm 20\%$ from its average value. This is a relatively weak weighting factor, compared with the effect of $\omega_{q,p}^2$.

We now write

$$\omega_{q,p}=c_{q,p}q, \tag{30}$$

and chose for $c_{q,p}$ the velocity of sound in the elastic limit (small q).

Although this is incorrect for large q, we are only concerned with indicating the relative importance of the longitudinal and transverse branches of the phonon spectrum and of the different directions of the same branch. In Table V, we list expressions for the velocity of sound squared times the density for the three principal directions[22] and evaluate these expressions using the published values of the elastic constants for potassium.[23] The table indicates how bad the assumption of an elastically isotropic solid, usually made in computing τ, is for the alkali metals. Since $c_{q,p}^2$ is substantially larger for the longitudinal modes than for the transverse ones the contribution of the transverse phonons to the integral Eq. (29) is weighted more strongly than that of the longitudinal ones. Likewise certain transitions, namely those using [110] phonons polarized 110 will be weighted much more strongly than others. The anisotropy in the sound velocity is averaged in Eq. (29), since transitions from a given initial state k to those states for which the transition probability is large involve many different phonon directions. Since the angular terms in the integral Eq. (29) weight certain values of the scattering angle, θ, heavily and since the phonon direction for fixed θ depends on the initial state $\mathbf{k}$ we do not expect averaging to be complete, although we expect the anisotropy of $\tau(\mathbf{k})$ to be considerably less than that of $c_{q,p}^2$.

Another source of anisotropy is the term $1/q^2$ in Eq. (29). For a normal (N) process, in which $K=0$, $\mathbf{q}$ depends only on the angle θ between $\mathbf{k}$ and $\mathbf{k}'$; how-

TABLE V. Velocity of sound in potassium. Numerical values are for potassium in units of dynes/cm$^2\times 10^{-10}$.

Direction of polarization	Values of $\rho c_{q,p}^2$ Direction of propagation 100	110	111
Longitudinal	$c_{11}=4.2$	$\frac{1}{2}[c_{11}+c_{12}+2c_{44}]=6.4$	$\frac{1}{3}[c_{11}+2c_{12}+4c_{44}]=7.1$
Transverse [001]	$c_{44}=2.6$	$c_{44}=2.6$	—
[110]	—	$\frac{c_{11}-c_{12}}{2}=0.41$	$\frac{c_{11}-c_{12}+c_{44}}{3}=1.15$

[21] J. M. Ziman, *Electrons and Phonons* (Oxford University Press, New York, 1960), p. 181.

[22] Jules de Launay, *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1956), p. 267.

[23] *American Institute of Physics Handbook* (McGraw-Hill Book Company, Inc., New York, 1957), pp. 3–81.